Dynamics of vesicles under stretching

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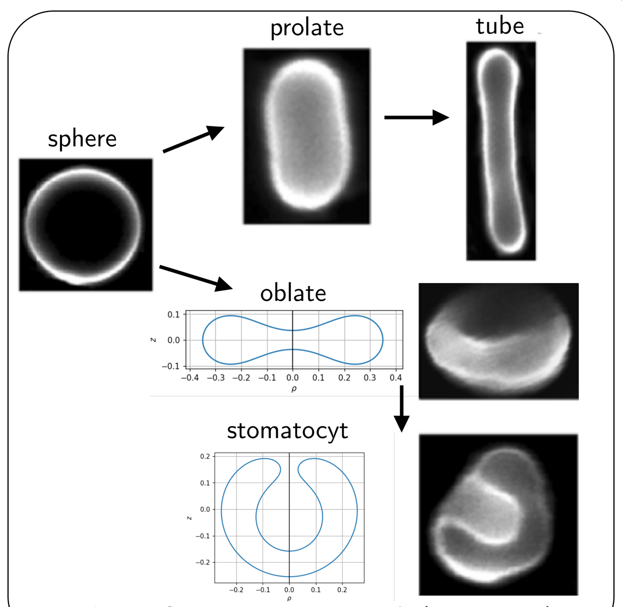
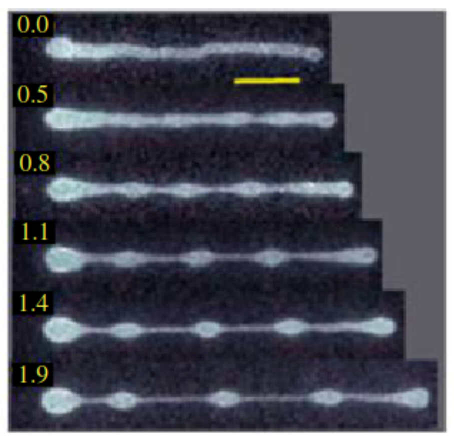
Introduction

Closed lipid membranes (vesicles), being flexible and incompressible, demonstrate a plethora of non-spherical shapes (especially under simple external influences). Within the framework of the hydrodynamic approach, the bilayer is treated as an infinitely thin liquid film with a surface density of free energy. The starting point for this consideration is Helfrich's energy [W. Helfrich, Z Naturforsch (1973)], which depends on vesicle shape. Corresponding surface forces cause flows in the viscous surrounding liquid. The study investigates the behavior of elongated vesicles under the influence of extending forces, such as uniaxial flow and optical tweezers. There are two critical values of force amplitude: beyond the first, a "dumbbell"-shaped structure forms with a possible infinite elongation, and the second is responsible for the so-called pearling instability, i.e., the formation of beads connected by thin tubes. We also provide a qualitative description of the phenomenon by discussing the variety of stretched shapes and conditions of quasi-stationarity and transition to infinite stretching. In addition, we study the highly nonlinear stage: a slow dynamics remains after the formation of pearls due to the thinness of tubes. Our results qualitatively agree with experimental observations [Kantsler, Segre, and Steinberg (2008)].



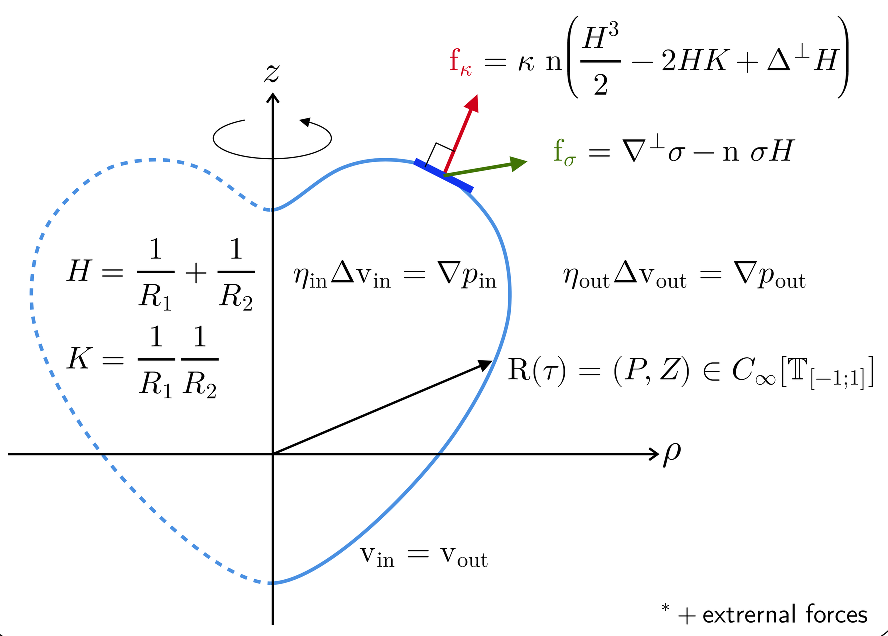
Shapes from Yanagisawa et al. (2008, PRL)



Tubular vesicle under pearling in elongation flow Kantsler V., Segre E., Steinberg V. (2008, PRL)

Govering equation

Stokes flow, surface forces

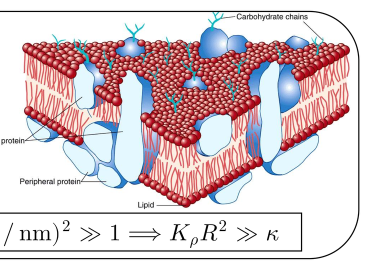


* + extrernal forces

Membrane energies

$$\mathcal{F}_\rho = \int \frac{K_\rho}{2}(n/n_0-1)^2 dS,\ K_\rho \sim 60\ \frac{k_BT}{\mathrm{nm}^2}$$

$$\mathcal{F}_\kappa = \frac{\kappa}{2}\int H^2 dS + \frac{\widetilde{\kappa}}{2}\int K dS,\ \ \kappa \sim 20\ k_BT$$



$$(R/\mathrm{nm})^2 \gg 1 \Longrightarrow K_\rho R^2 \gg \kappa$$

Use Green function of Stocks equation to determine velocity, induced by surface forces.

$$\mathrm{v} = \int_{S_1} \widehat{G}(\mathrm{f}_{\kappa 1} + \mathrm{f}_{\sigma 1}) \to v_\mu = \int_\tau \widetilde{G}_{\mu\nu}\ f_\nu d\tau$$

Surface tension is 'fast' variable - adiabatic approach

$$\begin{cases} \partial_t n = -\mathrm{v}\nabla n - n\nabla^\perp \mathrm{v} \\ \sigma \approx -K_\rho(n/n_0-1) \end{cases} \Longrightarrow \boxed{\nabla^\perp \mathrm{v} = 0}$$

Linear nonlocal equation for σ

$$\int_{S_1} (\nabla^\perp \widehat{G})\left[\nabla_1^\perp - \mathrm{n}_1 H_1\right]\sigma_1 = -\nabla^\perp \mathrm{v}_{-\sigma}$$

Energies are scaled by κ, surface tension σ by κ/R_0, velocity by $\kappa/\eta R_0^2$, time by $\eta R_0^3/\kappa$, where R_0 - appropriate lenght scale.

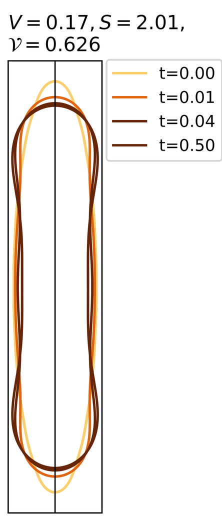
Final scheme $R \to f_\kappa \to \mathrm{v}_\kappa \to \nabla^\perp \mathrm{v} \to \sigma \to \mathrm{v}_\sigma \to \partial_t R$

Asymptotic prolate shapes

Conservation of volume and surface area, thus describing by reduced volume $\mathcal{V} = \dfrac{V/(4\pi/3)}{(S/4\pi)^{3/2}}$

Prolate vesicles have asymptotic shape: tube with universal edges:

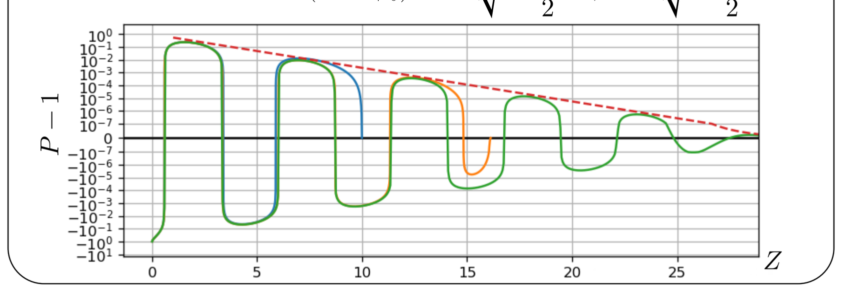
It can be determined as solution of force balance equation,where equlubrium pressure and surface tension can be found using balance far away from edge and scale invariance of Helfric energy.



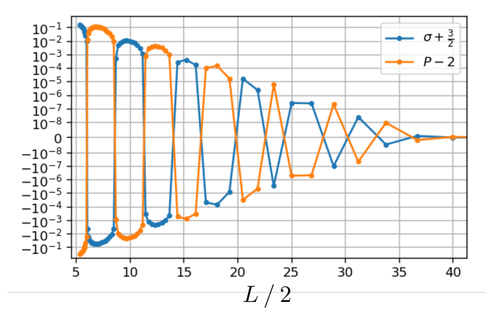
$$\begin{cases} \Delta P_{eq} = -\sigma_{eq} H + \kappa(H^3/2 - 2HK + \Delta^\perp H) \\ R \to R/\lambda \Longrightarrow \mathcal{F}_\kappa = \frac{\kappa}{2}\int H^2 dS \to \mathcal{F}_\kappa \end{cases} \Longrightarrow \boxed{\sigma_\infty = -\frac{3}{2}\frac{\kappa}{r_0^2},\ \ \Delta p_\infty = 2\frac{\kappa}{r_0^3}}$$

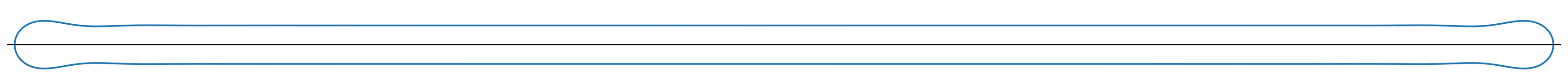
Far away from edge asymptotic:

$$P - 1 = \delta P \approx A e^{-\varkappa z}\sin(kz + \phi_0),\ \varkappa = \sqrt{\frac{\sqrt{3}-1}{2}},\ \ k = \sqrt{\frac{\sqrt{3}+1}{2}}$$



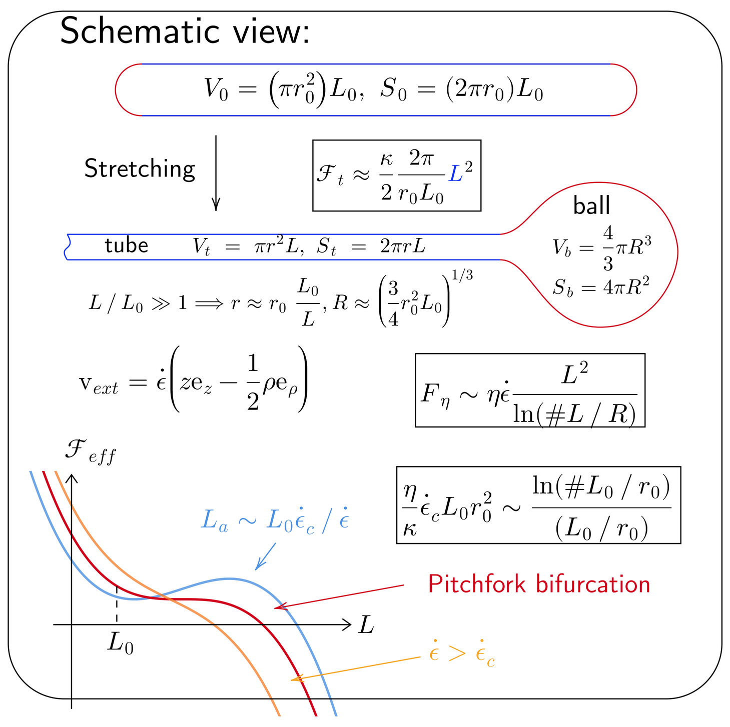
Convergence to the limit values





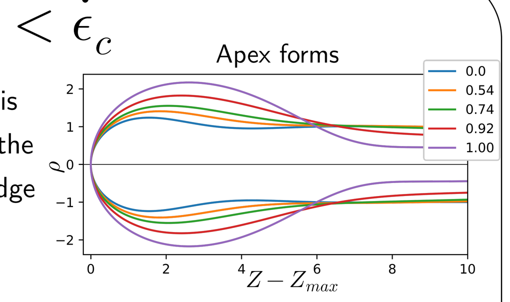
Infinite elongation transition in extansional flow

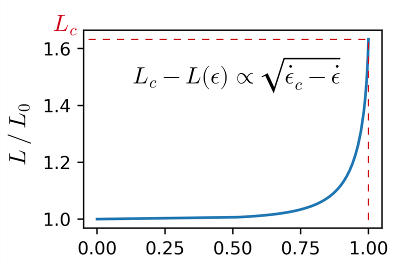
Schematic view:



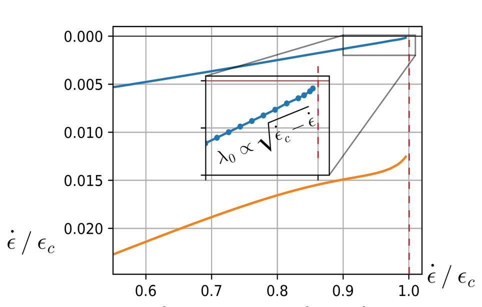
Under critical $\dot{\epsilon} < \dot{\epsilon}_c$

With a small rate, the vesicle is slightly deformed, while both the length and the shape of the edge change.





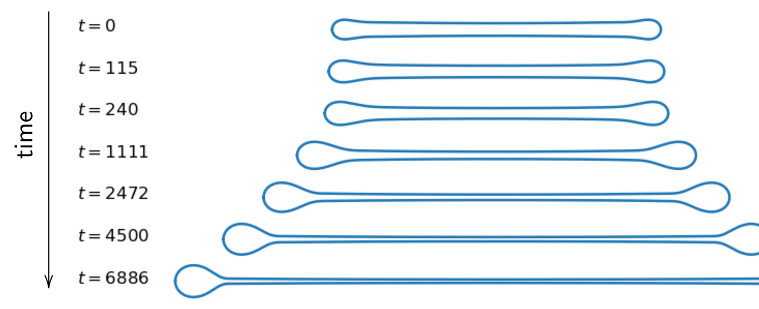
Steady-state elongation

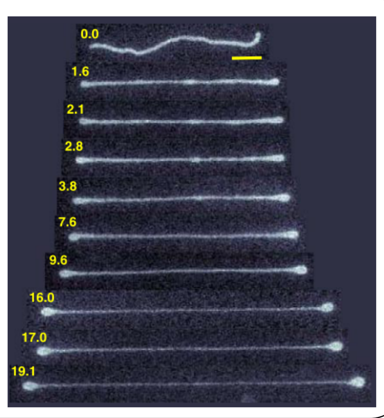


Two slowest normal modes $\lambda_{0,1}$

Upper critical $\dot{\epsilon} > \dot{\epsilon}_c$

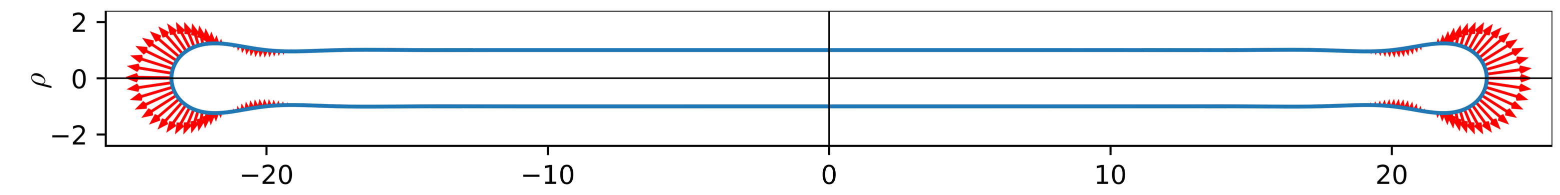
When the rate is above the critical value, the vesicle goes into unlimited stretching. On the right are the frames of our simulation and the experimental data from Kantsler V., Segre E., Steinberg V. (2008, PRL)





Pearling instability

Apply model normal elongating force $f = \gamma(H - H_0) \Longrightarrow$ same state is equlibrium with corrected surface tension $\delta\sigma = \gamma$

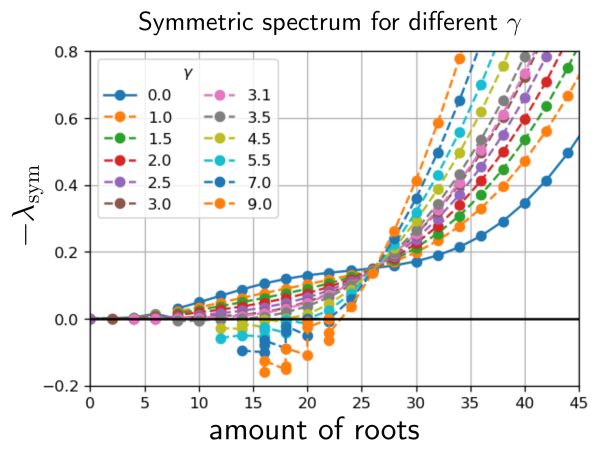


Linear stability theory

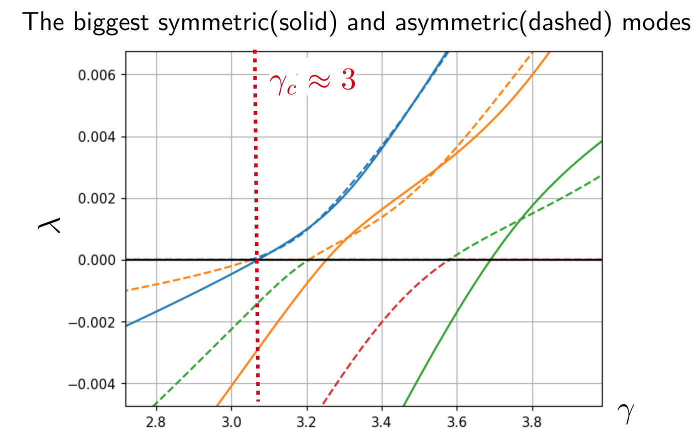
$$R = R_{\mathrm{eq}} + \delta R(t) \quad \partial_t R = F[R] \Longrightarrow \partial_t \delta R = \frac{\delta F}{\delta R}\delta R = \widehat{\Lambda}\delta R$$

Find normal modes $\widehat{\Lambda}\delta N_k = \lambda_k \delta N_k$ (Localy stable if $\forall k:\ \mathbb{R}\lambda_k < 0$)

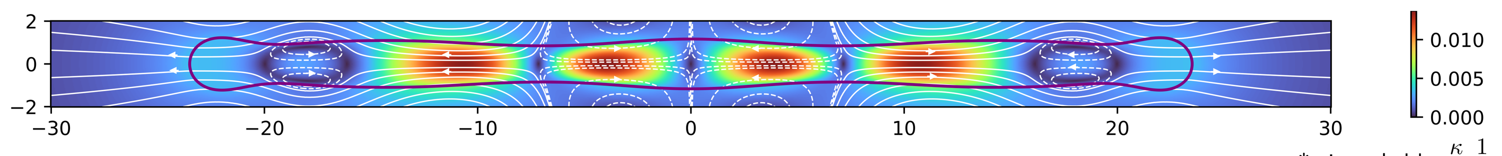
Symmetric spectrum for different γ



The biggest symmetric(solid) and asymmetric(dashed) modes

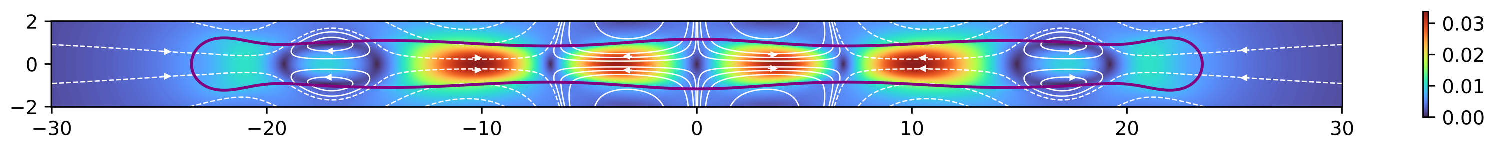


$\gamma > \gamma_c$ - pearls grow:



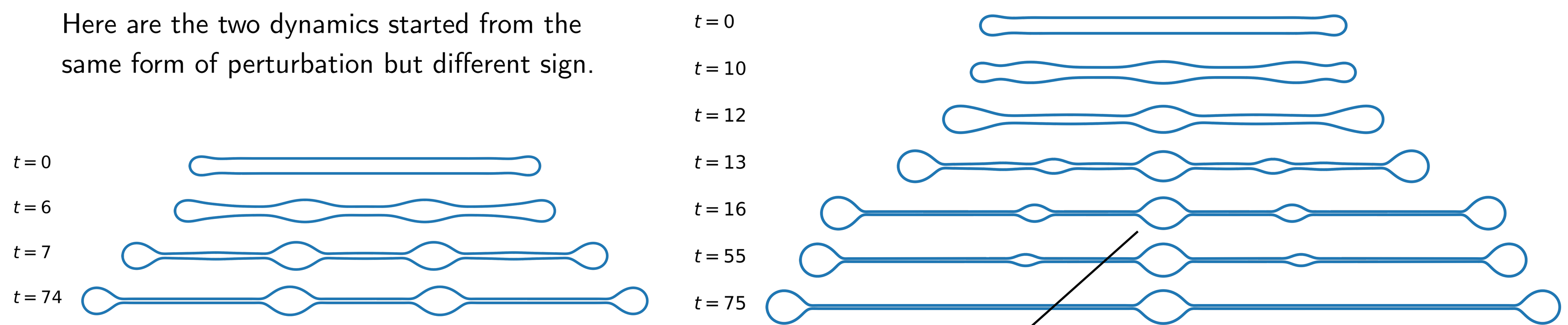
*v is scaled by $\frac{\kappa}{\eta}\frac{1}{R_e^2}$.

$\gamma < \gamma_c$ - pearls decrease:

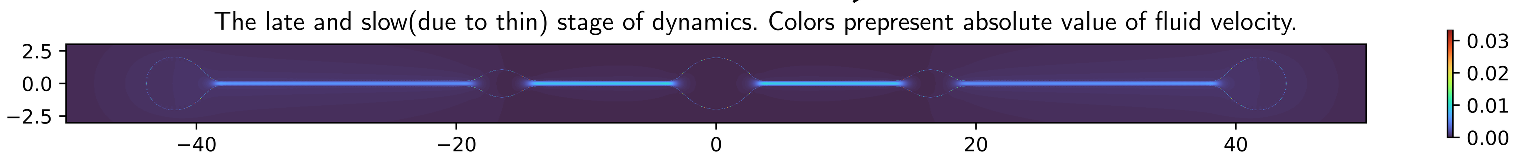


Development of the most unstable mode

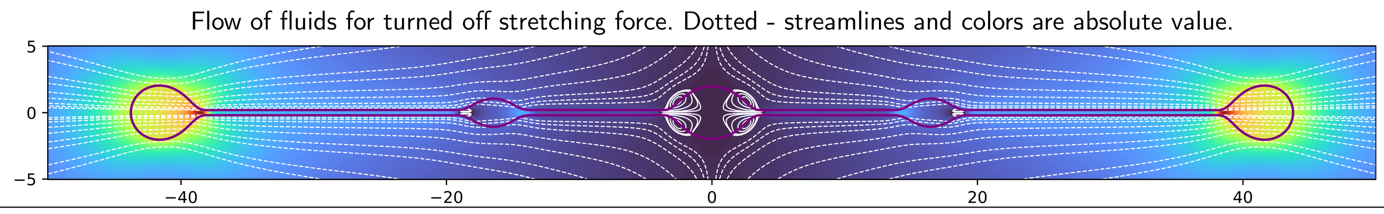
Here are the two dynamics started from the same form of perturbation but different sign.



The late and slow(due to thin) stage of dynamics. Colors prepresent absolute value of fluid velocity.



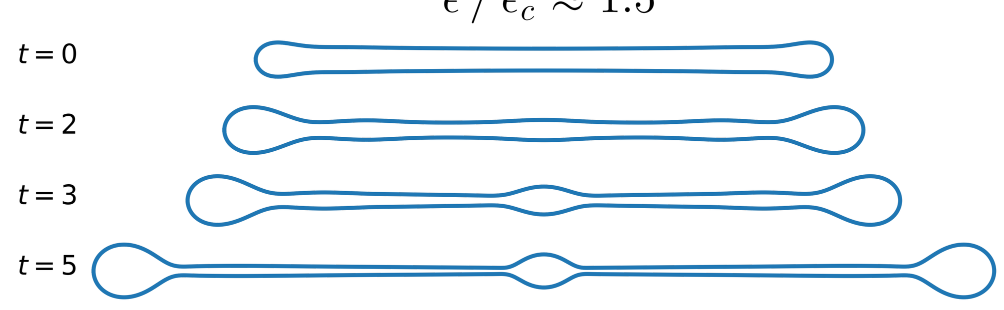
Flow of fluids for turned off stretching force. Dotted - streamlines and colors are absolute value.



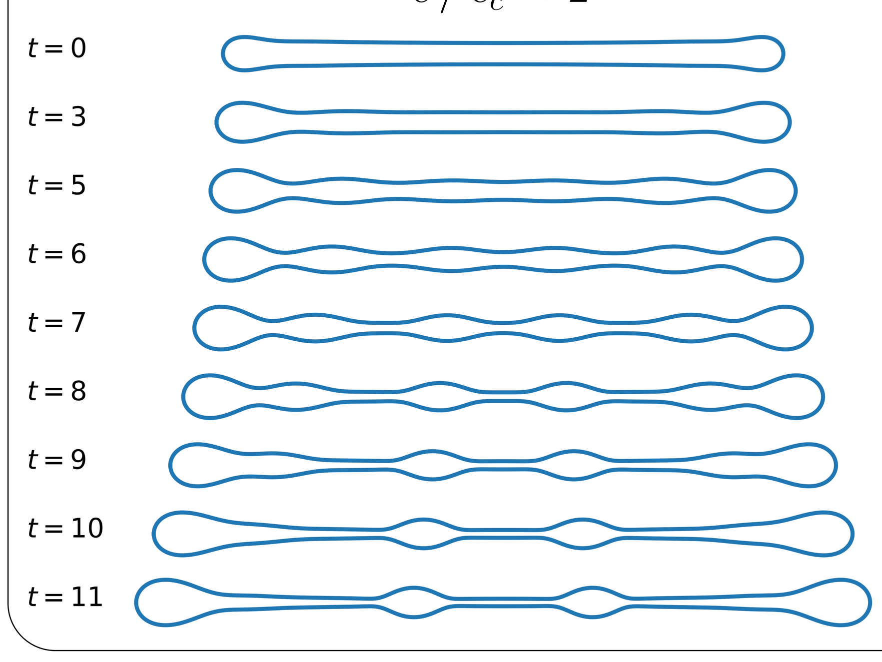
Instability in extensional flow $\mathrm{v}_{ext} = \dot{\epsilon}\left(z\mathrm{e}_z - \frac{1}{2}\rho\mathrm{e}_\rho\right)$

In the stretching flow, due to the presence of movement at the edges of the vesicle associated with infinite elongation, there is an effective disturbance for the central part of the cylinders, which propagates from the edges. The number of pearls depends on the rate of the supercritical flow.

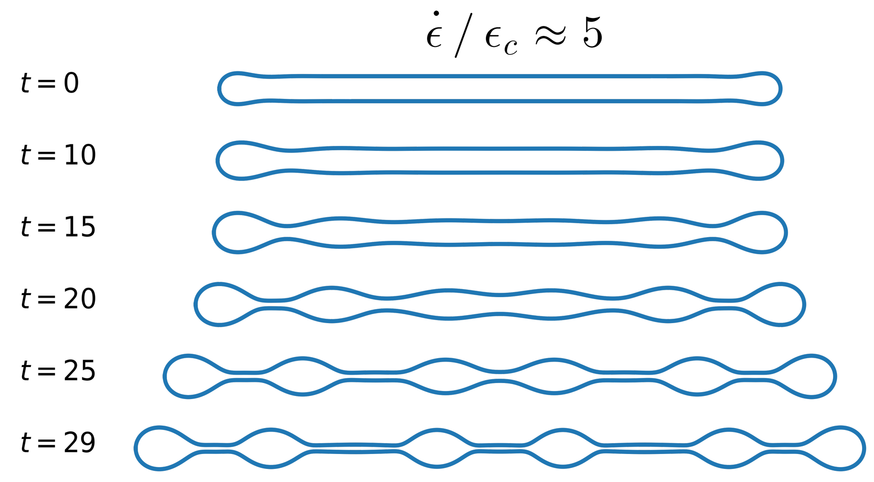
$\dot{\epsilon}/\epsilon_c \approx 1.5$



$\dot{\epsilon}/\epsilon_c \approx 2$



$\dot{\epsilon}/\epsilon_c \approx 5$



Conclusion

We developed a reliable algorithm for modeling the nonlinear dynamics of closed axially symmetric vesicles. Asymptotic shape and equilibrium parameters for elongated vesicles were derived. The infinit elongation transition in extansional flow were analized numerically and analitically. The "pearling instability" under the action of tensile forces applied to the edges of the vesicle and in strong extensional flow were considered; force critical values were obtained. We simulated the development of instability in a nonlinear regime with "pearls" formation and disappearing during slow late stage of dynamics.

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